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NPTEL ONLINE CERTIFICATION COURSE

Lecture 4: Basic Optics for Optical Sensing-II

Polarization & Scattering

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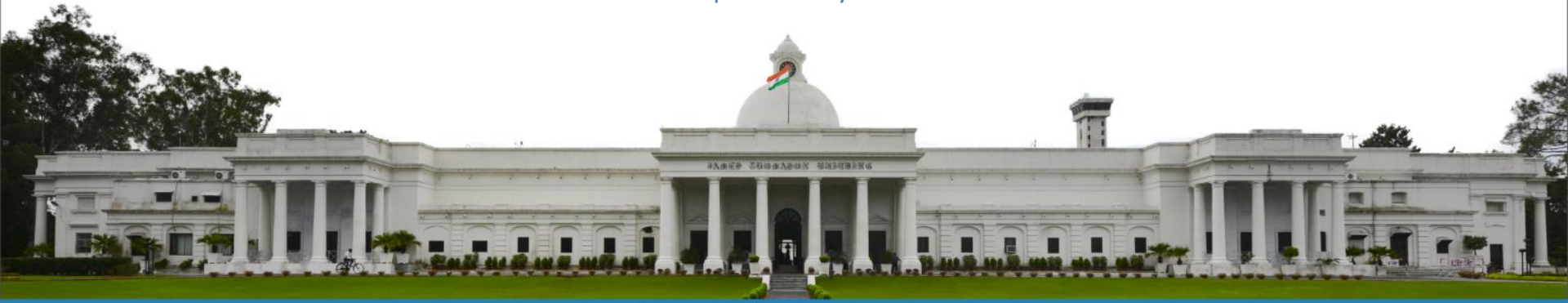
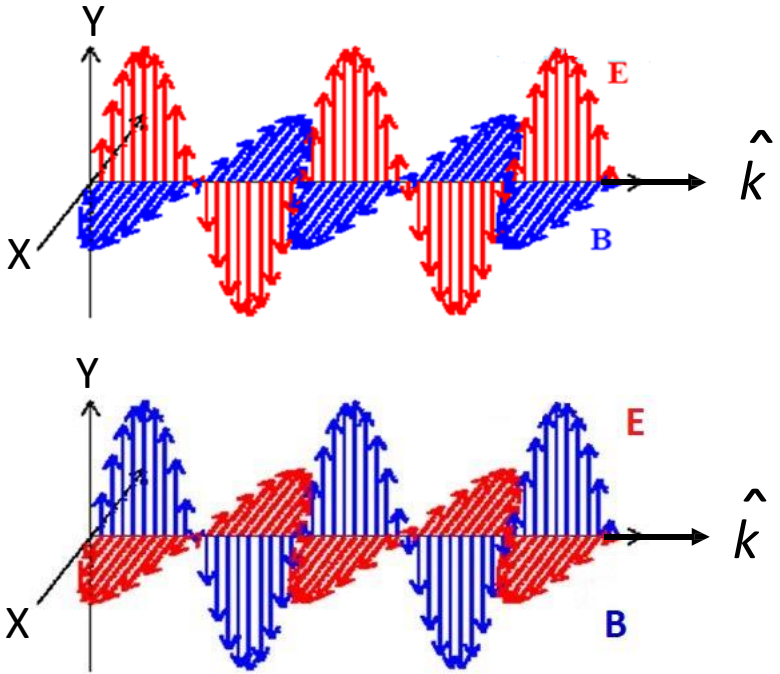


Polarization

For a given ***k*** there are two possible directions of ***E***

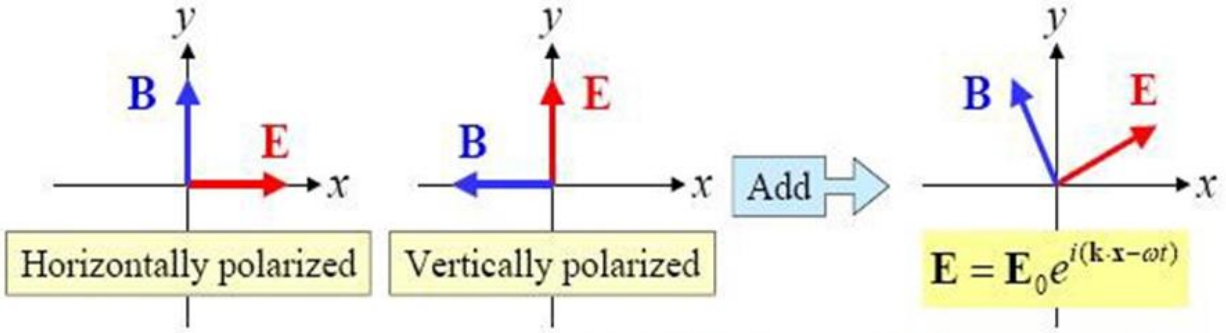
- ***E*** is either in x- or y- plane.
- *Direction of **E** is called the **polarization***



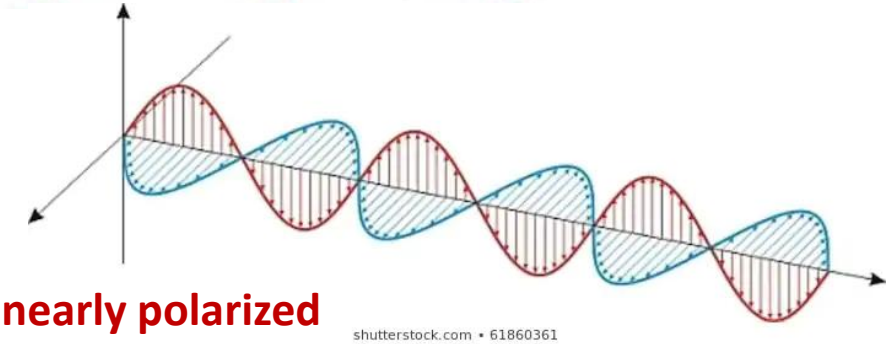
- ***They are TWO independent solutions of the wave equation***
- *Linear combinations make all the possible polaizations.*

Linear Polarization

- Looking up from downstream (+z)



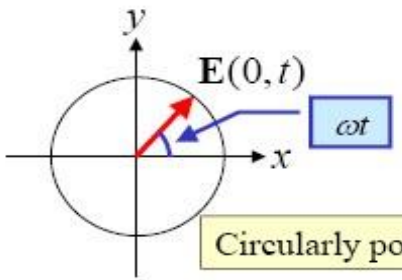
- Can make any angle from the horizontal and vertical polarized waves
- Can do more…

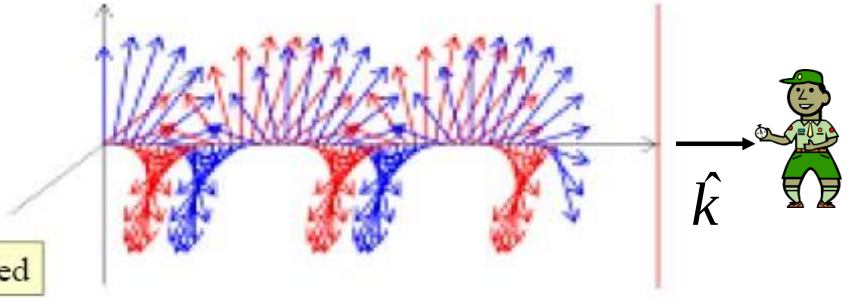


Linearly polarized

Circular Polarization

- We can use complex numbers for $\mathbf{E}_0$
  - Try, for example, $\mathbf{E}_0 = (E_0, iE_0)$ (phase difference of π/2)
  - $\Rightarrow \mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)} = E_0(e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}, ie^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)})$
  - Take the real part $\mathrm{Re}(\mathbf{E}) = E_0(\cos(\mathbf{k}\cdot\mathbf{x}-\omega t), -\sin(\mathbf{k}\cdot\mathbf{x}-\omega t))$
  - Consider $\mathbf{E}$ at the origin $\mathrm{Re}(\mathbf{E})_{\mathbf{x}=0} = E_0(\cos\omega t, \sin\omega t)$
    - $\mathbf{E}$ rotates around

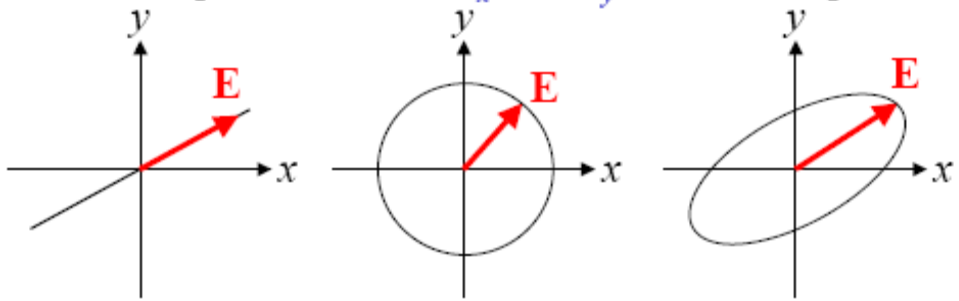




When looking opposite to the direction of propagation the E-vector for RCP rotates counterclockwise while for LCP, clockwise !

Elliptic Polarization

- General solution is a combination of linear and circular polarizations → Elliptic polarization
  - Linear polarization = E_x and E_y are in sync
  - Circular polarization = E_x and E_y are 90° off-phase



Polarization Ellipse

$$E_x = A_x \cos(\omega t - kz)$$
$$E_y = A_y \cos(\omega t - kz + \delta)$$

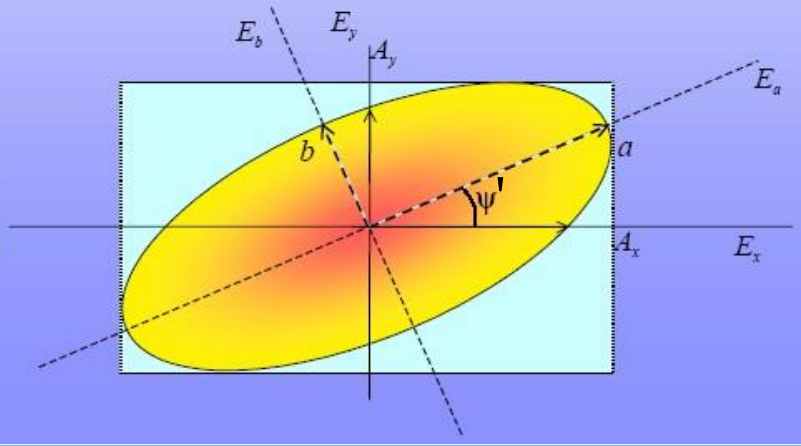
$$E_a = a \cos(\omega t - kz)$$
$$E_b = \pm b \sin(\omega t - kz)$$

$$\left(\frac{E_x}{A_x}\right)^2 - 2\frac{E_x E_y}{A_x A_y}\cos\delta + \left(\frac{E_y}{A_y}\right)^2 = \sin^2\delta$$

$$\left(\frac{E_a}{a}\right)^2 + \left(\frac{E_b}{b}\right)^2 = 1$$

$$\operatorname{tg}\chi = \pm\frac{b}{a}$$

A_x/A_y, δ



ψ', χ

Two Representations for Polarization

Either (ψ, δ)

Used in ellipsometry

$$\xi = \frac{E_y}{E_x} = \left|\frac{A_y}{A_x}\right| e^{i\delta} = |\xi| e^{i\delta} = \tan\psi \exp(i\delta)$$

Or (χ, ψ')

$$\tan(2\psi') = \frac{2\,\mathrm{Re}(\xi)}{1 - |\xi|^2}$$

ellipticity $e = \tan(\chi) = \pm b/a$

Note that $\sin(2e) = \dfrac{2\,\mathrm{Im}(\xi)}{1 + |\xi|^2}$

Circular, Elliptical and Linear Polarizations

Circular polarization

$\delta = \pm\frac{\pi}{2}$ $\qquad$ $\chi = \pm\frac{\pi}{4}$

$A_x = A_y$ $\qquad$ $a = b$

"–" left-, "+" right-handed $\qquad$ "–" right-, "+" left-handed

Elliptic polarization

$\delta\ (-\pi, \pi)$ $\qquad$ $\psi'\ (0, 2\pi)$

A_x / A_y $\qquad$ $\chi\ (-\pi/4, \pi/4)$

$\delta < 0 \ldots$ left $\qquad$ $\chi > 0 \ldots$ left

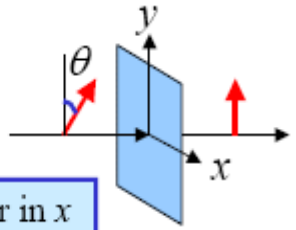
$\delta > 0 \ldots$ right $\qquad$ $\chi < 0 \ldots$ right

What is it for linear polarization ?

Linear Polarizers

Transmit one polarization and reject or absorb the other

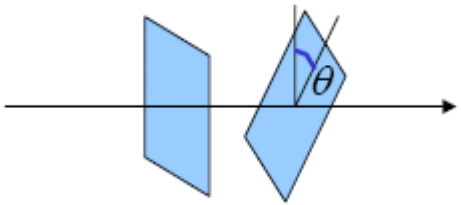
- If the polarization of incoming light is at an angle, $\mathbf{E}_{in} = E_{in}(\hat{\mathbf{y}}\cos\theta + \hat{\mathbf{x}}\sin\theta)$



unit vector in y | unit vector in x

$$\mathbf{E}_{out} = E_{in}\hat{\mathbf{y}}\cos\theta$$

  - Amplitude is $\times\cos\theta$ → Power is $P_{out} = P_{in}\cos^2\theta$ ← Malus' Law
- Easy to see using two filters



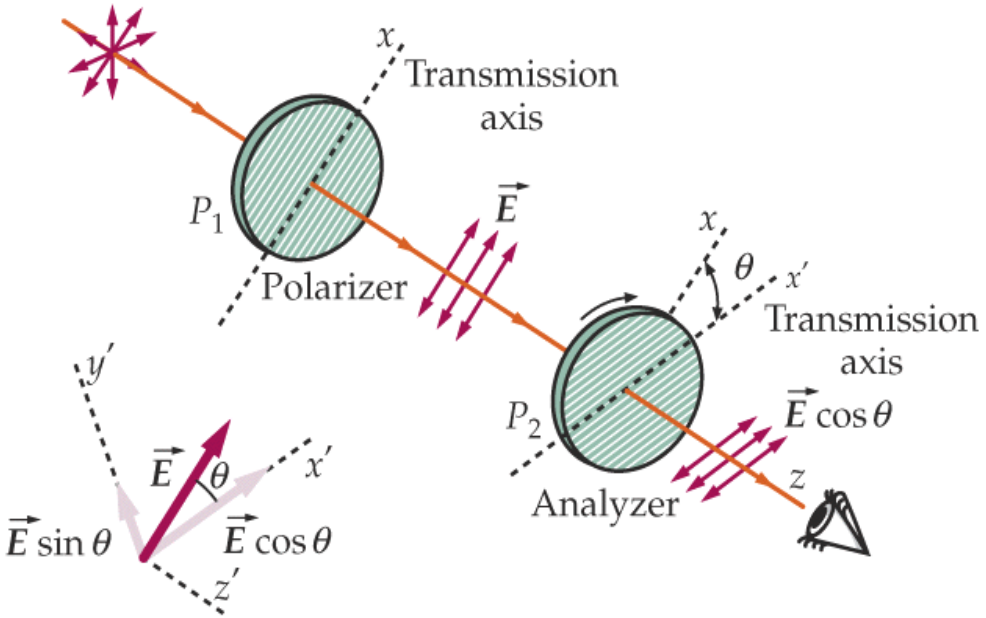
  - Light intensity falls as θ increases → 0 at $\theta = 90°$
  - What happens with 3 filters?

Two Polarizers – Physical Picture

This set of two linear polarizers produces LP (linearly polarized) light. What is the final intensity?

- P_1 transmits 1/2 of the unpolarized light: $I_1 = \frac{I_0}{2}$

- P_2 projects out the *E*- field component parallel to x' axis:

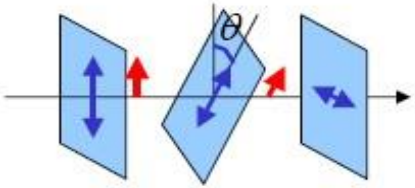


$$E_2 = E_1 \cos\theta$$

$$I \propto E^2 \Rightarrow I_2 = I_1 \cos^2\theta = \frac{1}{2} I_0 \cos^2\theta$$ = 0 if $\theta = \pi/2$ (i.e., crossed)

Stack of Polarizers

- Result with 3 polarizing filters a bit surprising
  - Filters 1 + 3 block all light
  - Adding 2 makes some light pass!



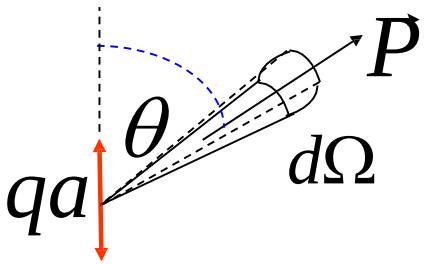
- Nothing really to be surprised
  - After filter 1, $\mathbf{E}_1 = E_1\hat{\mathbf{y}}$
  - After filter 2, $\mathbf{E}_2 = E_1 \cos\theta($
  - After filter 3, $\mathbf{E}_3 = E_1\hat{\mathbf{x}}\cos\theta\sin\theta$

$$= \frac{E_1 \sin 2\theta}{2}\hat{\mathbf{x}}$$

Maximum intensity is 25% at 45°

Prove that when many polarizers are inserted between the external crossed ones so that each with small angle ϑ with respect to the other, total transmission is obtained !

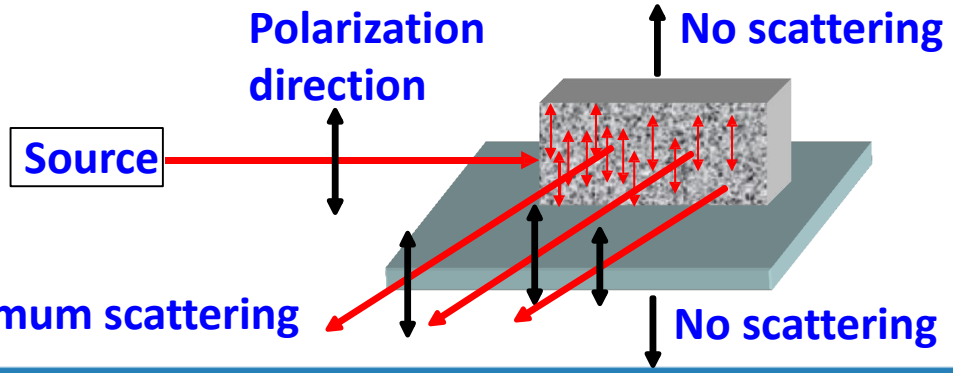
Polarization and Scattering - Dipole Radiation

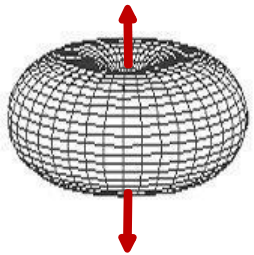


$$\frac{dP}{d\Omega} = \frac{q^2 a^2 \omega^4}{4\pi c^3} \sin^2\theta \propto \frac{\sin^2\theta}{\lambda^4}$$

Hence a dipole does not radiate along its axis !

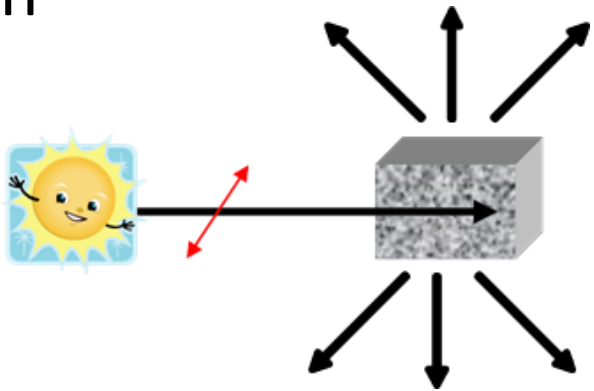
Polarization dependence because the induced dipoles direction depends on the light polarization !





Field distribution in 3D

Sky Polarization

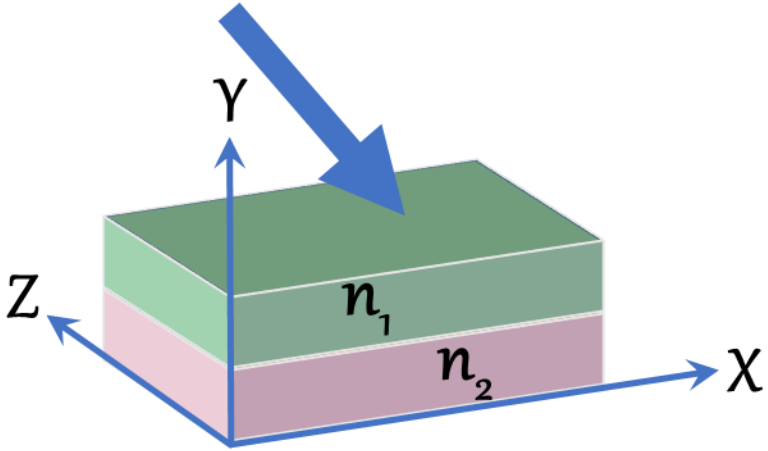


Scattering is perpendicular to induced dipoles, that is to incident polarization

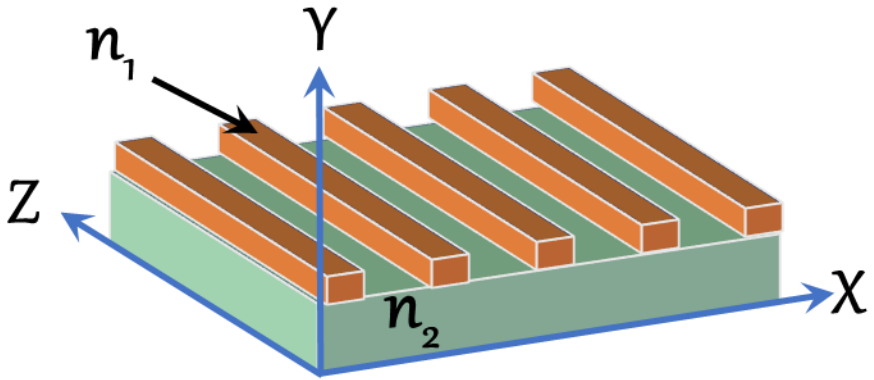
This photo, taken with a 20mm wide-angle lens with linear polarizer, shows the high degree of linear polarization of the sky 90 degrees away from the sun. The region is so well- defined that the approximate solar altitude can be inferred from the tilt of the dark region of high polarization.

Transverse Electric (TE) and Transverse Magnetic (TM) Polarizations

What will be the Transverse Electric (TE) and Transverse Magnetic (TM) Components for the following configurations?



(a)



(b)

Summary

1. Polarization of light plays an important role in direction of propagation and scattering.
2. It is useful in determining the optical properties of any configuration.

Thank You